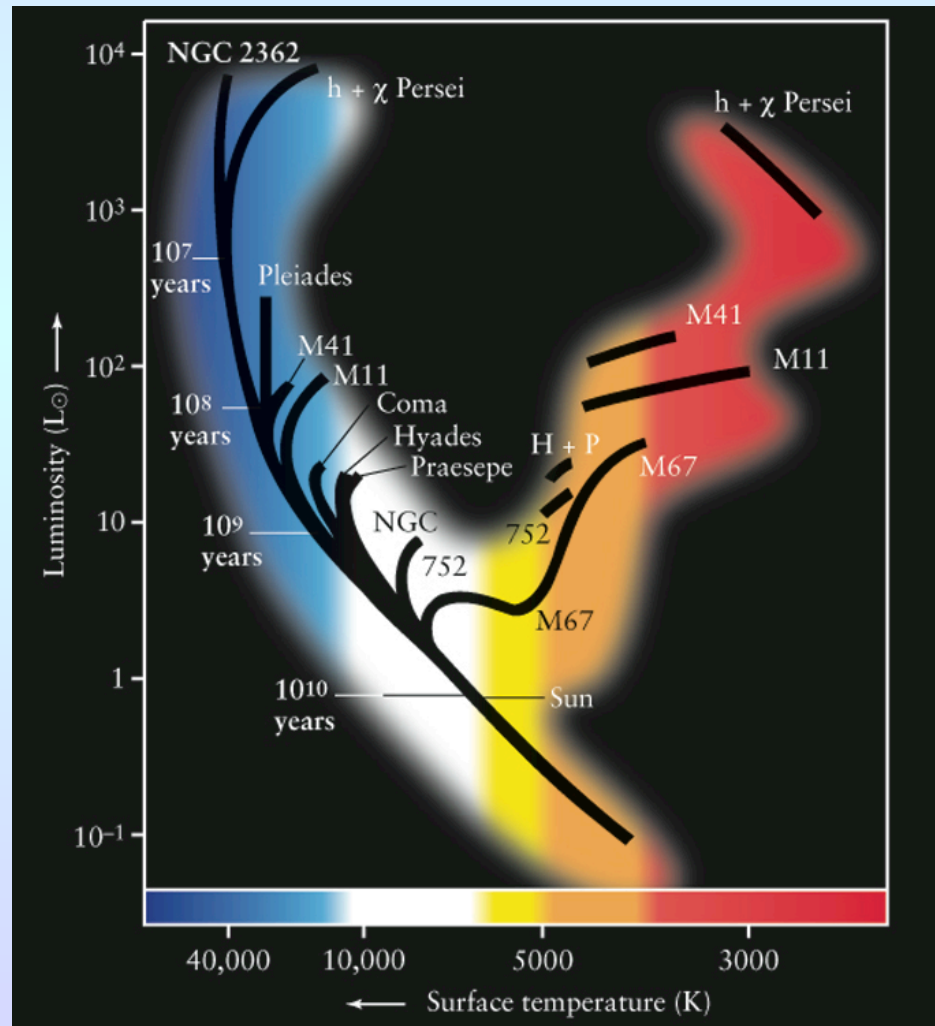
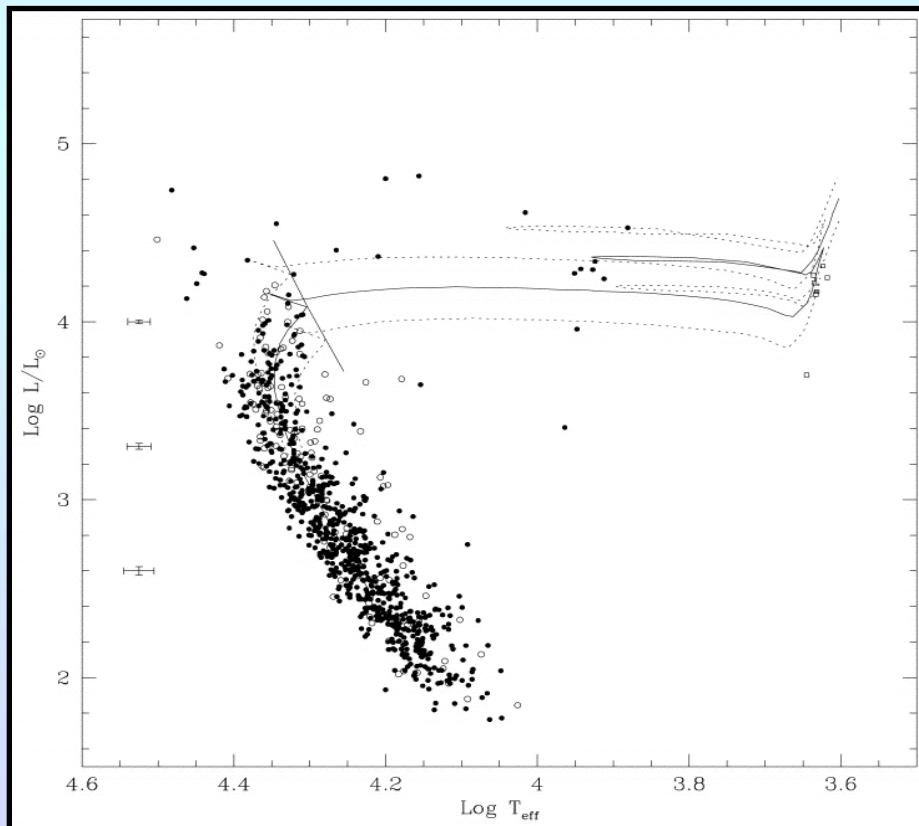


# Star Clusters

If a set of stars are all born at the same time, then the age of this “Simple Stellar Population” (SSP) is determined by the main-sequence “turnoff”.

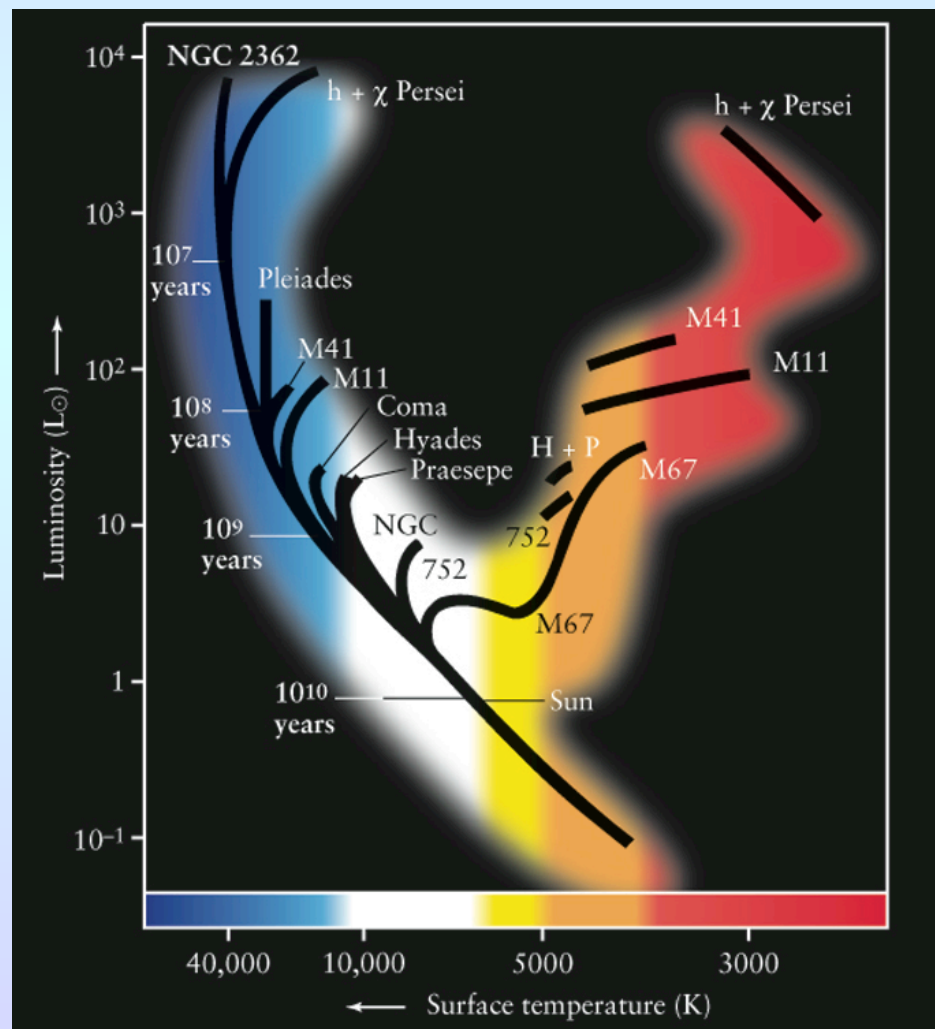
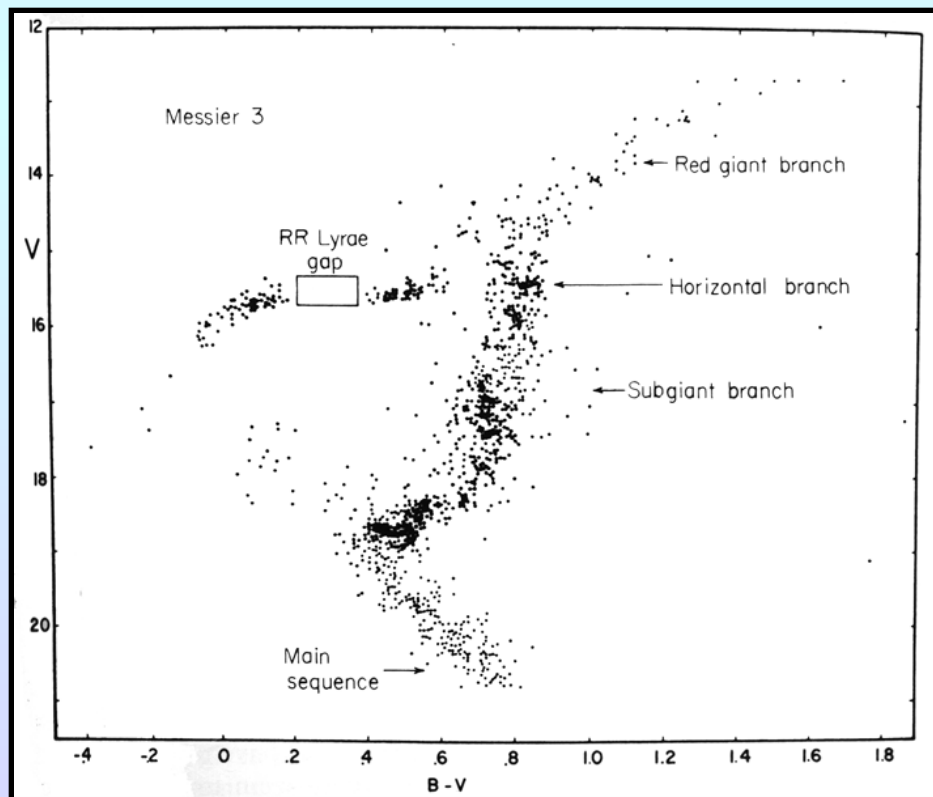
$$\tau \propto \frac{M}{L} \propto \frac{M}{M^\eta} \propto M^{1-\eta} \propto M^{-2.5}$$



# Star Clusters

If a set of stars are all born at the same time, then the age of this “Simple Stellar Population” (SSP) is determined by the main-sequence “turnoff”.

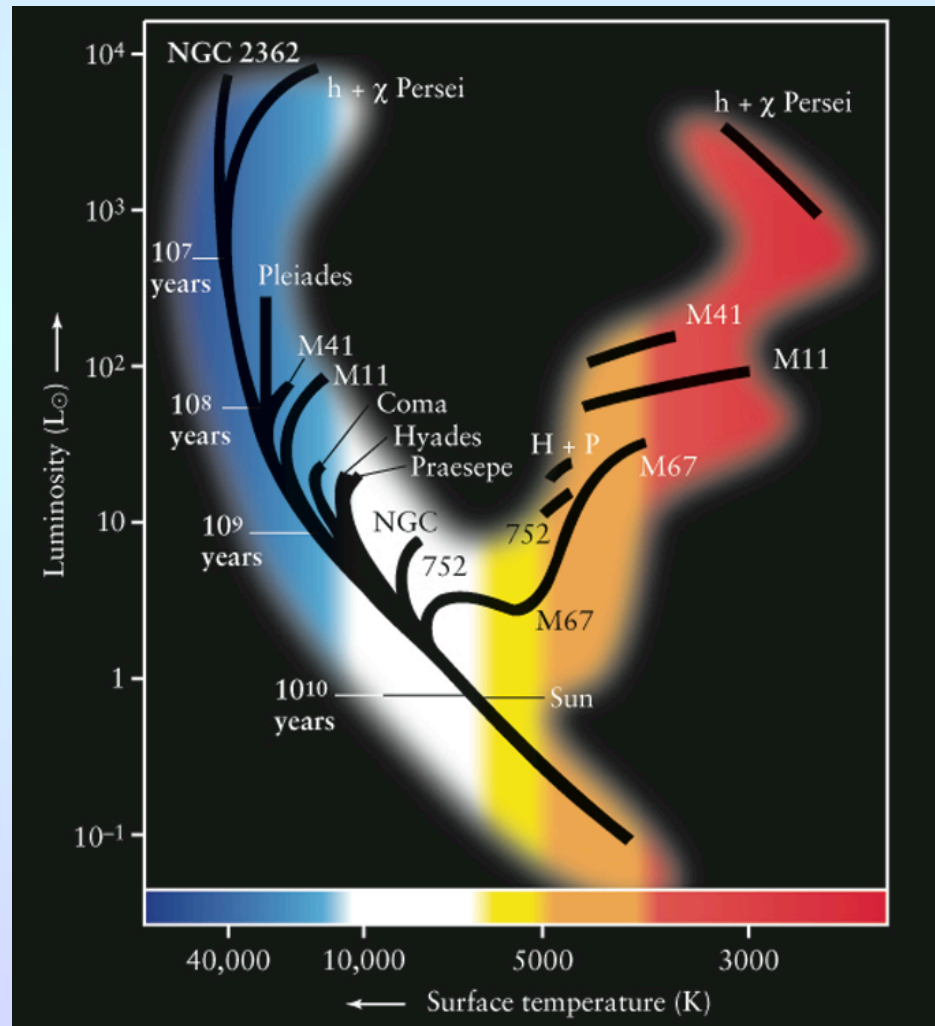
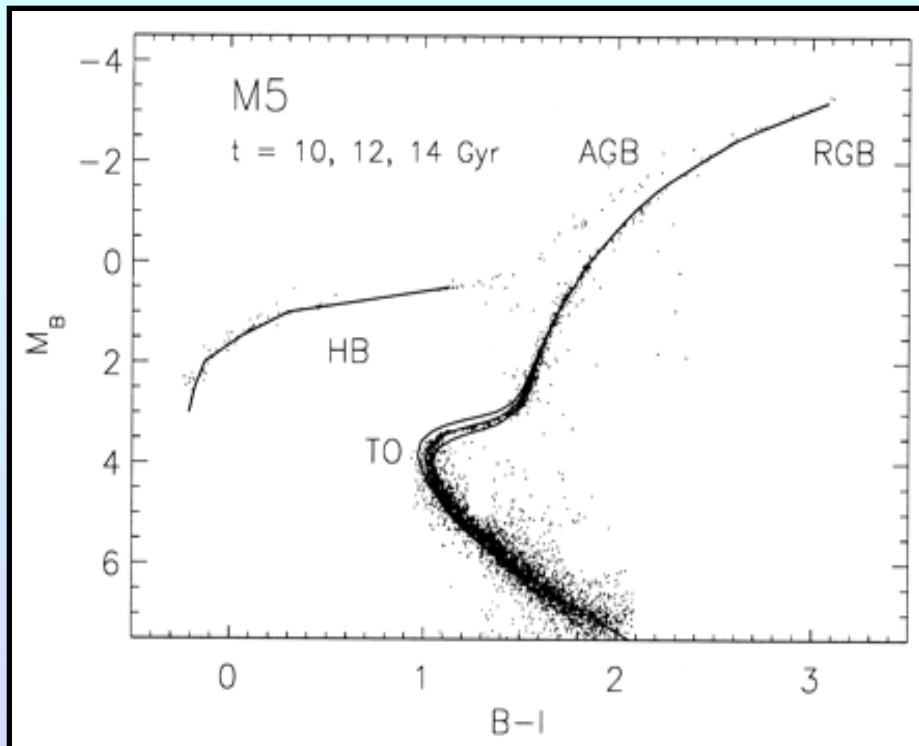
$$\tau \propto \frac{M}{L} \propto \frac{M}{M^\eta} \propto M^{1-\eta} \propto M^{-2.5}$$



# Star Clusters

If a set of stars are all born at the same time, then the age of this “Simple Stellar Population” (SSP) is determined by the main-sequence “turnoff”.

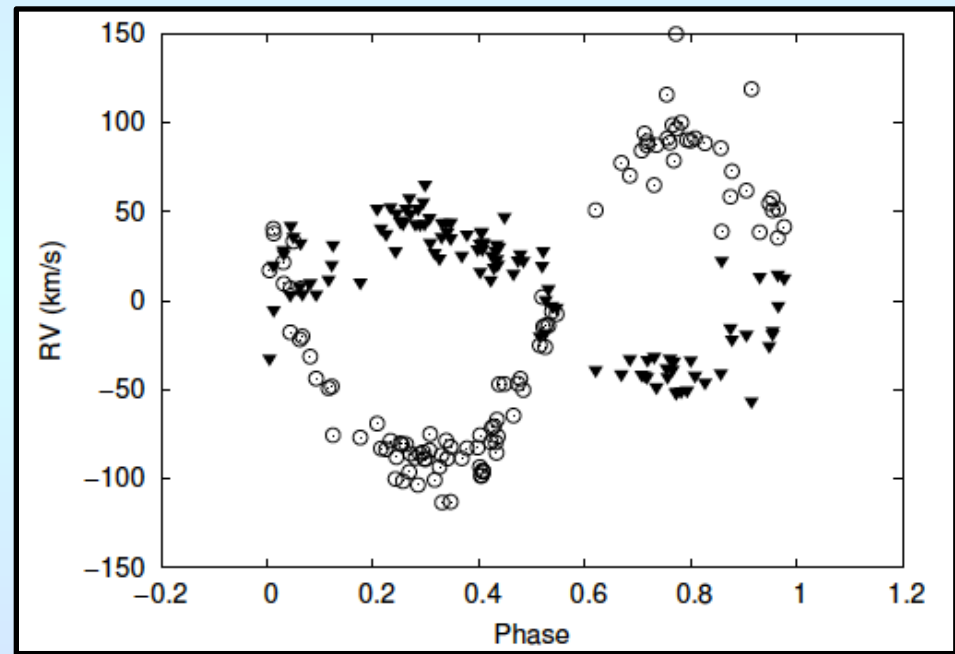
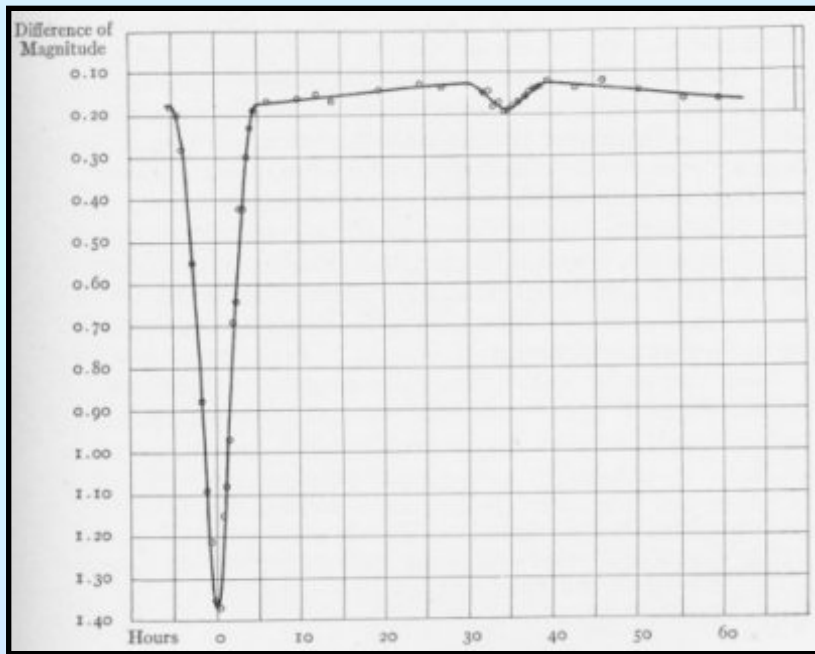
$$\tau \propto \frac{M}{L} \propto \frac{M}{M^\eta} \propto M^{1-\eta} \propto M^{-2.5}$$



# Problems for Stellar Evolution

Most problems involving stellar structure and evolution are now reasonably well understood. But what about:

- The Algol paradox ( $\beta$  Persei, a 2.867 day eclipsing, spectroscopic binary at a distance of 22 pc)



Algol A is a  $3.5 M_{\odot}$  B main sequence star.

Algol B is  $0.8 M_{\odot}$  K giant star.

How can the lower mass star be more evolved?

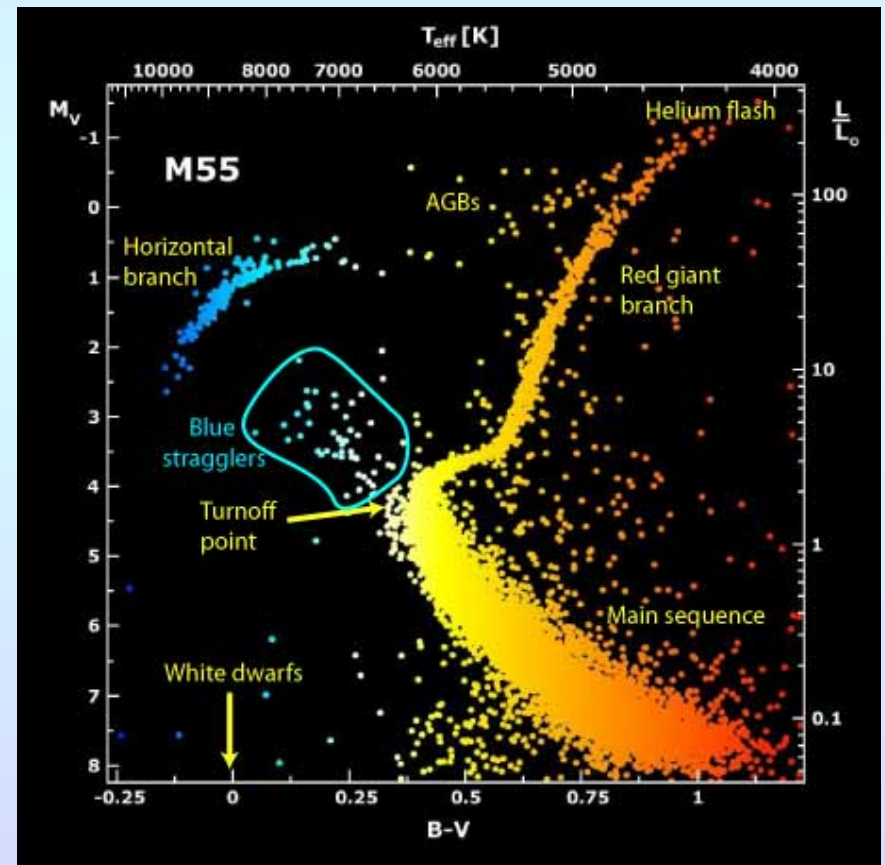
# Problems for Stellar Evolution

Most problems involving stellar structure and evolution are now reasonably well understood. But what about:

- The Algol paradox ( $\beta$  Persei, a 2.867 day eclipsing, spectroscopic binary at a distance of 22 pc)
- Blue Straggler Stars

“Main sequence” stars that appear to have lifetimes that are much shorter than the age of their cluster.

Why are they still there?

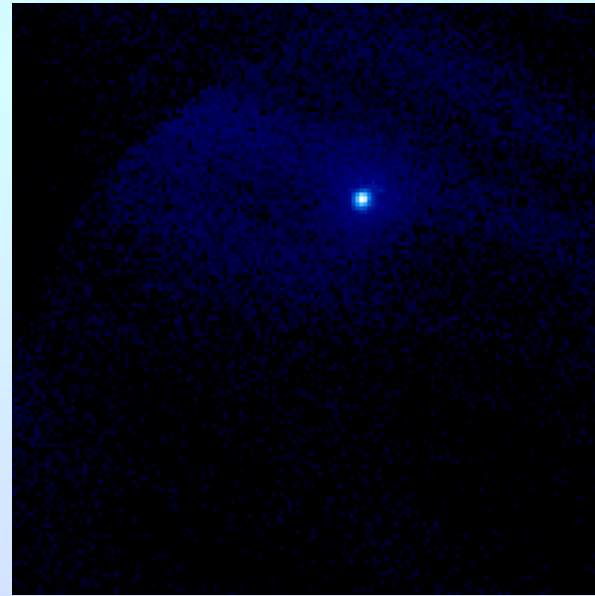
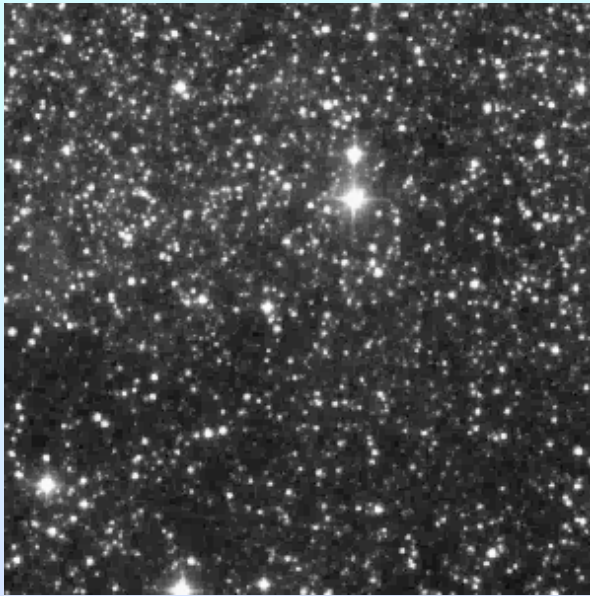




# Problems for Stellar Evolution

Most problems involving stellar structure and evolution are now reasonably well understood. But what about:

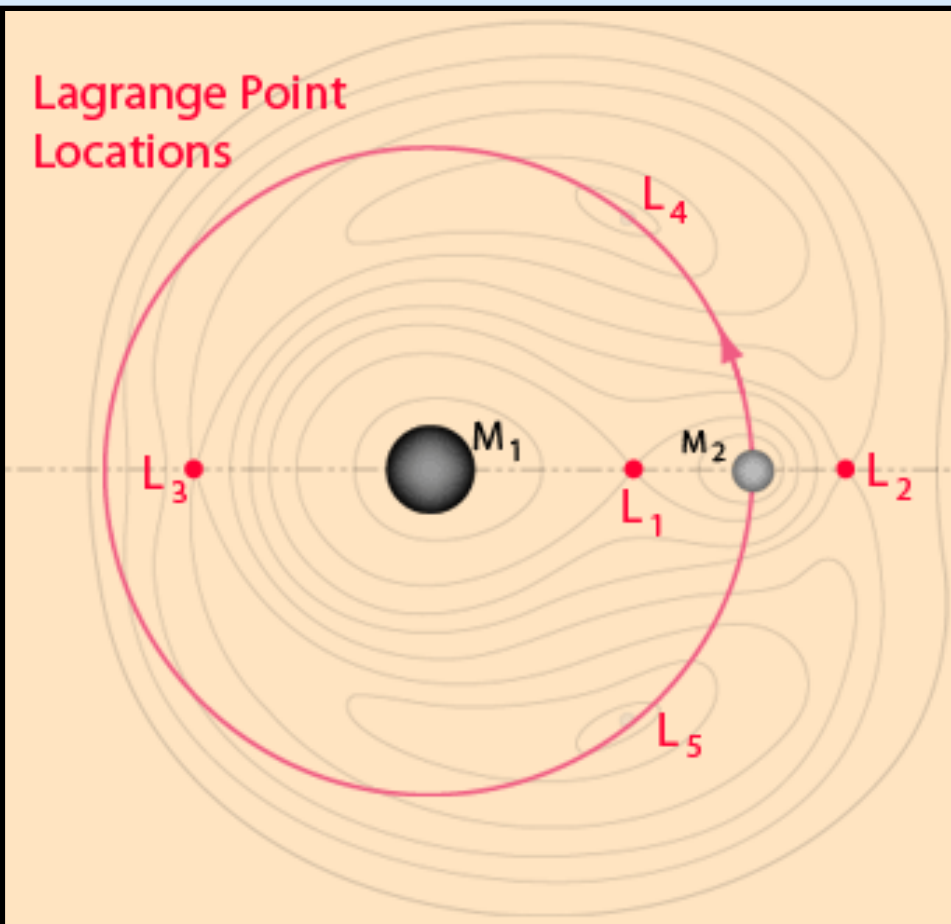
- The Algol paradox ( $\beta$  Persei, a 2.867 day eclipsing, spectroscopic binary at a distance of 22 pc)
- Blue Straggler Stars
- X-ray Sources



Even the hottest ( $\sim 100,000^\circ$  K) stars should emit a negligible amount of X-rays. Where do these X-rays come from?

# The Roche Potential

The answer to these puzzles involves mass transfer between stars. In the rotating coordinate system, the gravitational plus centrifugal potential is



$$\Phi(r) = -\frac{GM_1}{|r - r_1|} - \frac{GM_2}{|r - r_2|} - \frac{1}{2}(\omega \times r)^2$$

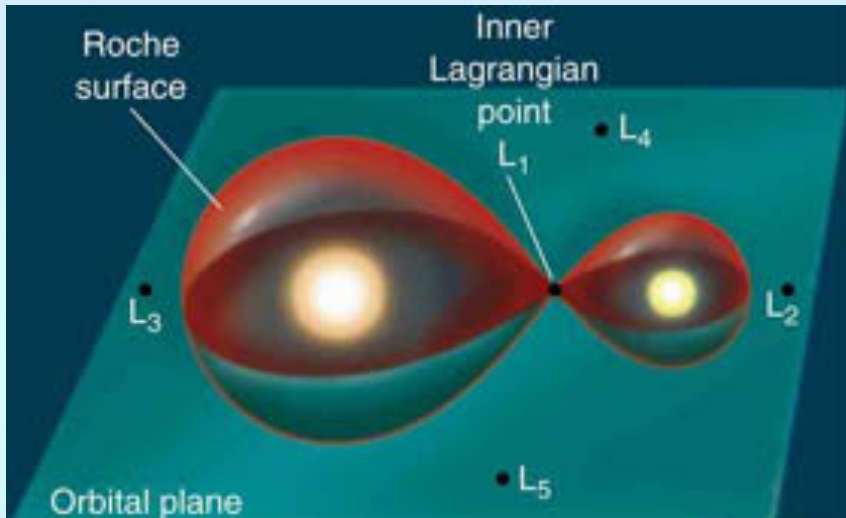
where from Kepler's 3<sup>rd</sup> law,

$$\omega = \sqrt{\frac{G(M_1 + M_2)}{a^3}} \hat{\mathbf{n}}$$

Note that the lobes are not circular, and exact solutions must (usually) be solved numerically. But there are approximations.

# The Roche Potential

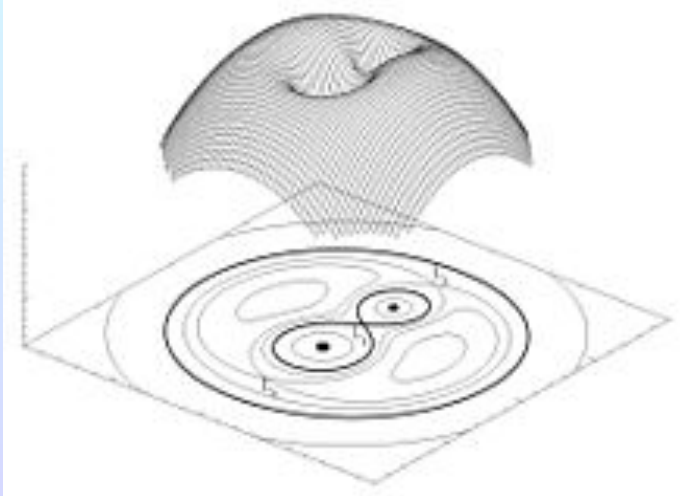
The answer to these puzzles involves mass transfer between stars. In the rotating coordinate system, the gravitational plus centrifugal potential is



$$\Phi(r) = -\frac{GM_1}{|r - r_1|} - \frac{GM_2}{|r - r_2|} - \frac{1}{2}(\omega \times r)^2$$

where from Kepler's 3<sup>rd</sup> law,

$$\omega = \sqrt{\frac{G(M_1 + M_2)}{a^3}} \hat{\mathbf{n}}$$



Note that the lobes are not circular, and exact solutions must (usually) be solved numerically. But there are approximations.



# Size of the Roche Lobe

Let  $q = M_2 / M_1$  be the mass ratio (with  $0.1 < q < 10$ ). The Roche Lobe “radius” (where the volume of the Roche lobe equals that of a sphere) is usually approximated (to better than  $\sim 2\%$ ) with either

$$\frac{R_L}{a} = 0.462 \left( \frac{q}{1+q} \right)^{1/3} \quad \text{for } q < 0.8 \quad (\text{Paczynski 1971})$$

or

$$\frac{R_L}{a} = \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})} \quad \text{for any } q \quad (\text{Eggleton 1983})$$

If a star expands to exceed its Roche Lobe (or if, for some other reason, the semi-major axis of the orbit shrinks), mass transfer will occur. This mass transfer can change the Roche Lobe sizes and drive further mass loss.

# Response to Mass Transfer

How will a system respond to mass transfer? There are 3 factors to consider:

- 1) The response of the orbit's semi-major axis to mass loss
- 2) The response of the star's Roche Lobe ( $R_L$ ) to mass loss
- 3) The response of the star's radius to mass loss

# Response of the Semi-Major Axis

Assume that star  $M_2$  is losing mass (so  $\dot{M}_2 < 0$ ). Now look at the orbital angular momentum of the system

$$\begin{aligned} J &= (M_1 \omega a_1^2 + M_2 \omega a_2^2) (1 - \varepsilon^2)^{1/2} \\ &= \left\{ M_1 \omega \left( \frac{M_2}{M_1 + M_2} \right)^2 a^2 + M_2 \omega \left( \frac{M_1}{M_1 + M_2} \right)^2 a^2 \right\} (1 - \varepsilon^2)^{1/2} \\ &= \frac{M_1 M_2}{M_1 + M_2} \omega a^2 (1 - \varepsilon^2)^{1/2} = \frac{M_1 M_2}{M_T} \left( \frac{2\pi}{P} \right) a^2 (1 - \varepsilon^2)^{1/2} \\ J &= M_1 M_2 \left\{ \frac{G a}{M_T} (1 - \varepsilon^2) \right\}^{1/2} \quad (\text{From Kepler's 3rd law}), \text{ so} \end{aligned}$$

$$\ln J = \ln M_1 + \ln M_2 - \frac{1}{2} \ln M_T + \frac{1}{2} \ln a + \frac{1}{2} \ln G + \frac{1}{2} \ln (1 - \varepsilon^2)$$

Thus, as mass transfer proceeds, the semi-major axis must change as

$$\frac{\dot{a}}{a} = \frac{\dot{M}_T}{M_T} + 2 \frac{\dot{J}}{J} - 2 \left( \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} \right) - \frac{1}{2} \frac{2 \varepsilon \dot{\varepsilon}}{(1 - \varepsilon^2)}$$

# Response of the Semi-Major Axis

Assume that star  $M_2$  is losing mass (so  $\dot{M}_2 < 0$ ). Now look at the orbital angular momentum of the system

$$\begin{aligned} J &= \left( M_1 \omega a_1^2 + M_2 \omega a_2^2 \right) \\ &= \left\{ M_1 \omega \left( \frac{M_2}{M_1 + M_2} \right)^2 a^2 + M_2 \omega \left( \frac{M_1}{M_1 + M_2} \right)^2 a^2 \right\} \\ &= \frac{M_1 M_2}{M_1 + M_2} \omega a^2 = \frac{M_1 M_2}{M_T} \left( \frac{2\pi}{P} \right) a^2 \\ J &= M_1 M_2 \left\{ \frac{G a}{M_T} \right\}^{1/2} \quad \text{(From Kepler's 3<sup>rd</sup> law), so} \end{aligned}$$

$$\ln J = \ln M_1 + \ln M_2 - \frac{1}{2} \ln M_T + \frac{1}{2} \ln a + \frac{1}{2} \ln G$$

Thus, as mass transfer proceeds, the semi-major axis must change as

$$\frac{\dot{a}}{a} = \frac{\dot{M}_T}{M_T} + 2 \frac{\dot{J}}{J} - 2 \left( \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} \right)$$

# Response of $R_L$ to Mass Transfer

Now, we can look at how the Roche Lobe radius of the stars will change. Recall that  $q = M_2 / M_1 = M_2 / (M_T - M_2)$ . If we apply one of the approximations for Roche Lobe size, i.e.,

$$\frac{R_L}{a} = 0.462 \left( \frac{q}{1+q} \right)^{1/3} \Rightarrow \frac{\dot{R}_L}{R_L} = \frac{\dot{a}}{a} + \frac{1}{3} \frac{\dot{M}_2}{M_2} - \frac{1}{3} \frac{\dot{M}_T}{M_T}$$

then

$$\frac{\dot{R}_L}{R_L} = \frac{2}{3} \frac{\dot{M}_T}{M_T} + 2 \frac{\dot{J}}{J} - \frac{5}{3} \frac{\dot{M}_2}{M_2} - 2 \frac{\dot{M}_1}{M_1} - \frac{1}{2} \frac{2\varepsilon \dot{\varepsilon}}{(1-\varepsilon^2)}$$



# Response of $R_L$ to Mass Transfer

We can now define two types of mass transfer: conservative, where  $\dot{M}_T = 0$ ,  $\dot{J} = 0$ , and  $\dot{M}_2 = -\dot{M}_1$ , and non-conservative, where mass may be lost from the system, and the orbital angular momentum may change. Reasons for the latter include

- Mass loss, due to winds, etc.
- Magnetic breaking from the stellar **B** fields
- Gravitational Radiation

If the mass loss is conservative (and  $\varepsilon = 0$ ), then the orbit will shrink if

$$\frac{\dot{a}}{a} = \frac{\dot{M}_T}{M_T} + 2 \frac{\dot{J}}{J} - 2 \left( \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} \right) - \frac{1}{2} \frac{2\varepsilon \dot{\varepsilon}}{(1-\varepsilon^2)} = -2 \frac{\dot{M}_2}{M_2} \left( 1 - \frac{M_2}{M_1} \right) \Rightarrow \frac{M_2}{M_1} > 1$$

and the Roche Lobe will shrink if

$$\frac{\dot{R}_L}{R_L} = -2 \frac{\dot{M}_2}{M_2} \left( 1 - \frac{M_2}{M_1} \right) + \frac{1}{3} \frac{\dot{M}_2}{M_2} = -2 \frac{\dot{M}_2}{M_2} \left( \frac{5}{6} - \frac{M_2}{M_1} \right) \Rightarrow \frac{M_2}{M_1} > \frac{5}{6}$$

# Response of Stellar Radius to Mass Loss

As mass is lost, a star's radius will adjust structure. If we define

$$\zeta_L = \frac{d \ln R_L}{d \ln M} \quad \text{and} \quad \zeta = \frac{d \ln R}{d \ln M}$$

Then the result of the mass loss will depend on the sign of  $\zeta$  and how rapidly the star can adjust its structure. This usually depends on the state of the outer layers of the star. Notes:

- For many stars,  $\zeta > 0$ , but degenerate stars and stars with deep convective envelopes have  $R \propto M^{-1/3}$ , (i.e.,  $\zeta = -0.33$ ) Mass loss involving stars with deep convective layers will produce a runaway.
- Stars will react in multiple ways to mass loss; there is a  $\zeta_{\text{dyn}}$ ,  $\zeta_{\text{KH}}$ , and  $\zeta_{\text{nuc}}$ .

# Types of Mass Loss

In general, there are 3 possibilities for mass loss:

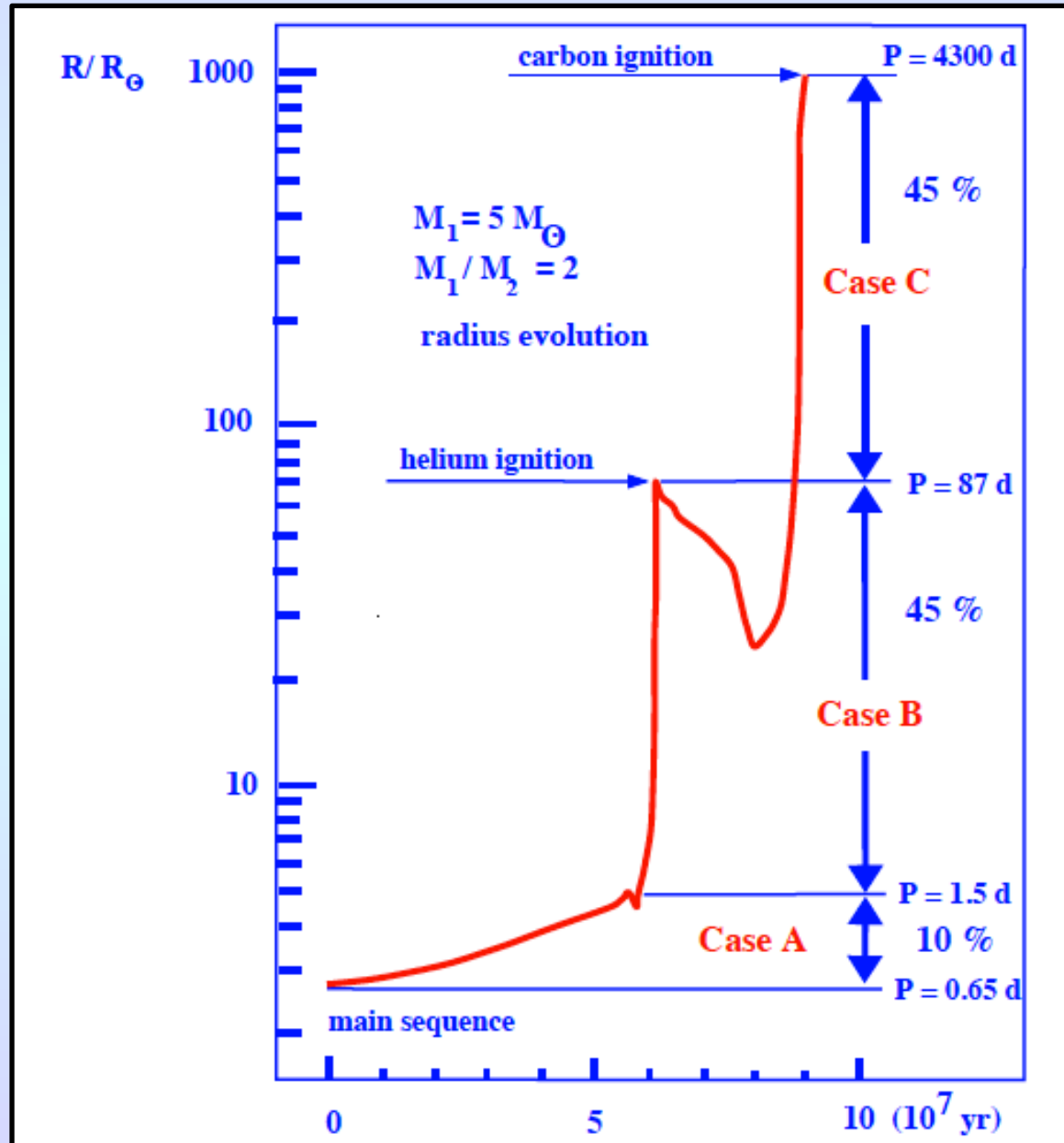
- $\zeta_L > \zeta_{\text{dyn}}$ : the star cannot remain within its Roche Lobe under the conditions of hydrostatic equilibrium; mass is lost extremely rapidly (on a dynamical timescale) and the star becomes part of a common envelope system. This occurs for stars with deep convective envelopes (red giants), very low mass (fully convective) main sequence stars, and white dwarfs.
- $\zeta_{\text{dyn}} > \zeta_L > \zeta_{\text{KH}}$ : hydrostatic equilibrium is maintained throughout the mass transfer process, but the star cannot maintain thermal equilibrium. The star remains just filling its Roche Lobe, and relaxation towards thermal equilibrium drives the mass loss. This occurs for high-mass main sequence stars.
- $\zeta_{\text{KH}} > \zeta_L$ : the star is in hydrostatic and thermal equilibrium; if mass loss continues, it must do it on a nuclear timescale (i.e., slow expansion of a star on the main sequence).

# Case A, B, and C Mass Transfer

Case A: mass transfer while on the main sequence.

Case B: mass transfer on the giant branch (before Helium ignition)

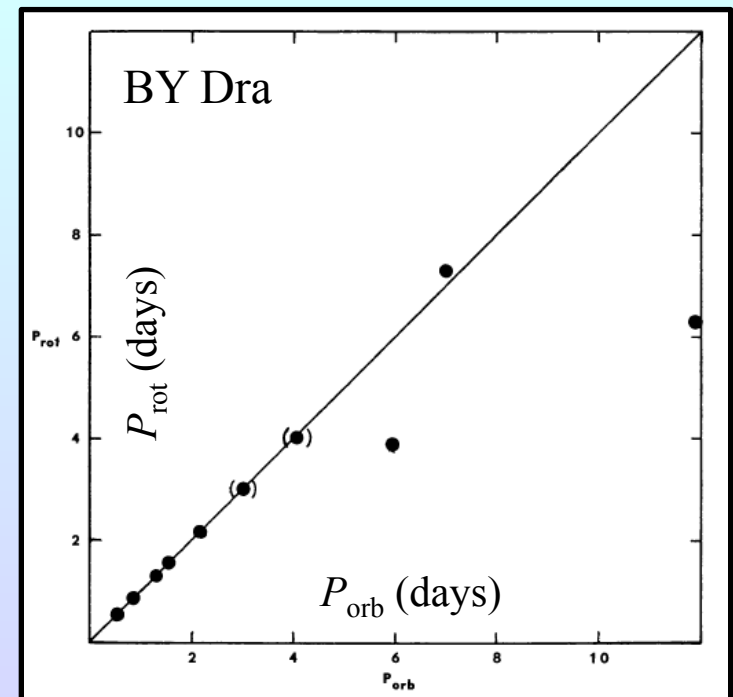
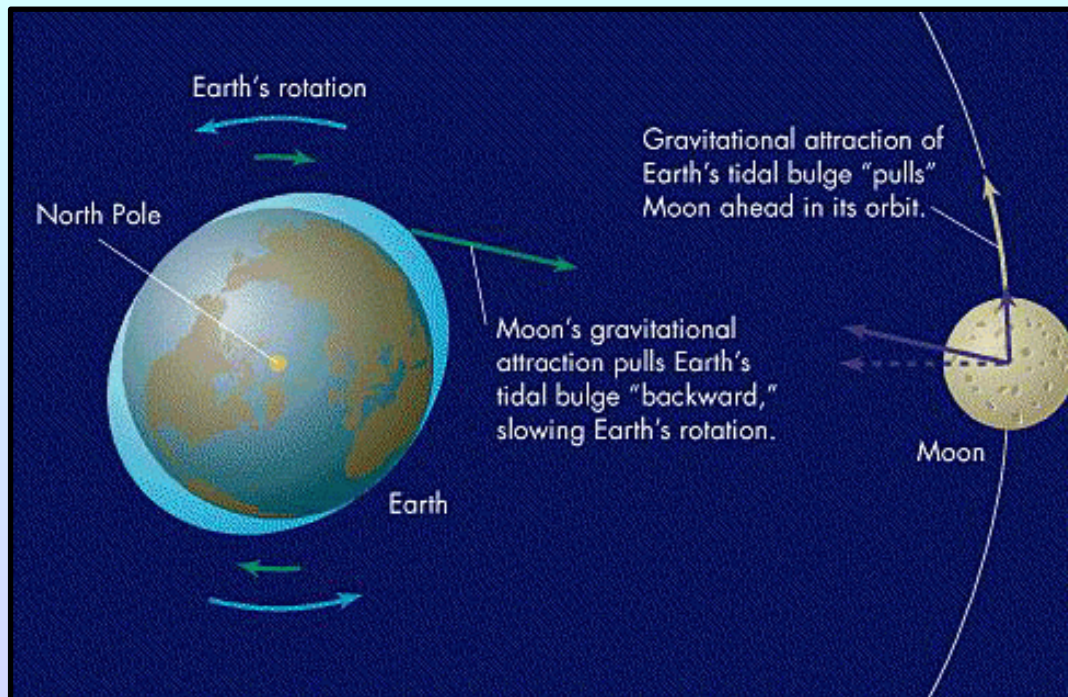
Case C: mass transfer on the asymptotic giant branch (before Carbon ignition)



# Tides and Spin Up

Tides can transfer orbital angular momentum into rotational angular momentum (and vice versa). For example, the Earth is spinning down, while the Moon's orbit is expanding. For close binary systems,  $P_{\text{rot}} > P_{\text{orbit}}$ , so the stars are spun up until a tidal lock is achieved.

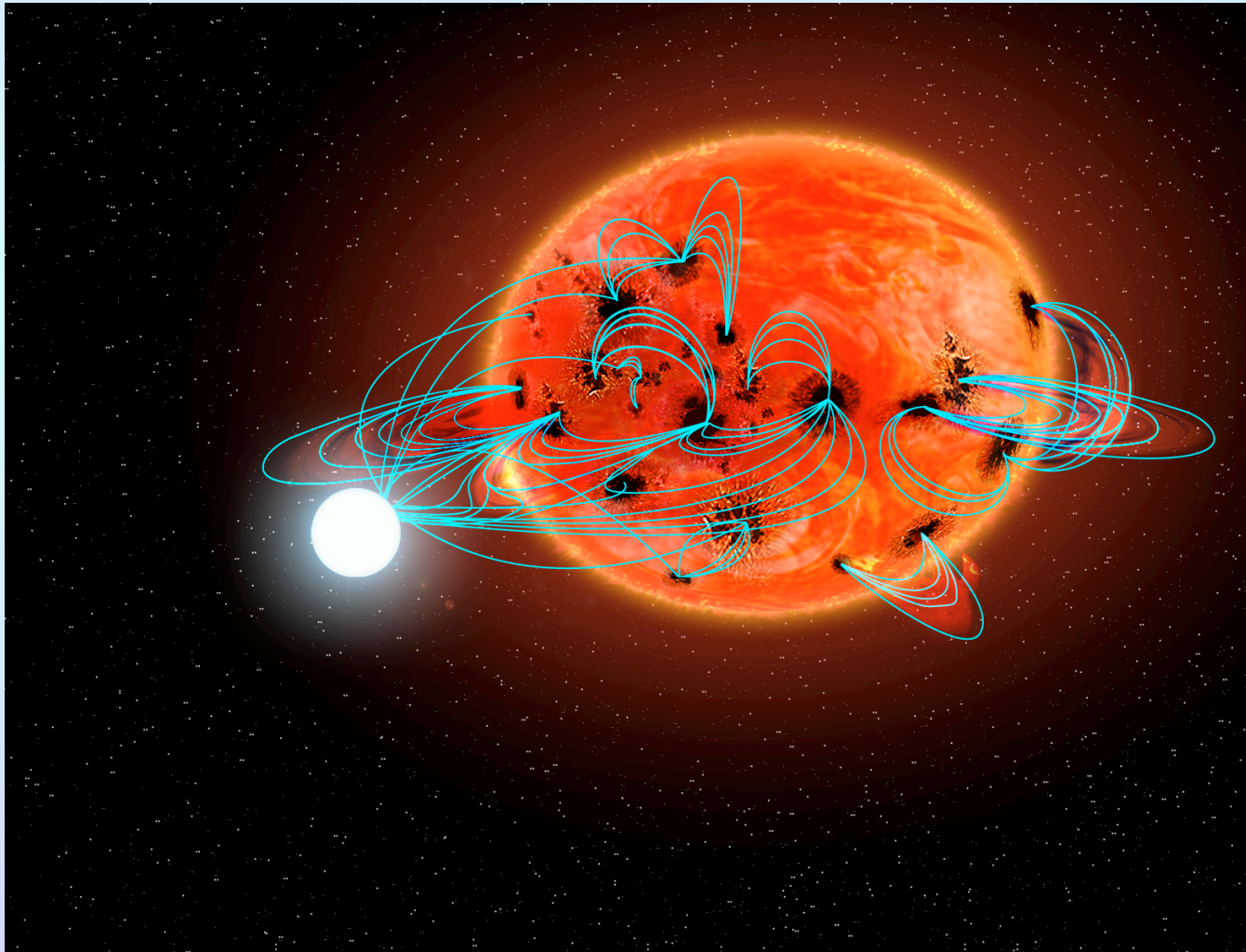
$$F_{\text{grav}} = -\frac{GMm}{r^2} \Rightarrow F_{\text{tide}} = \left( \frac{dF_{\text{grav}}}{dr} \right) dr = \frac{2GMm}{r^3} dr$$





# Tidal Locking and Stellar Activity

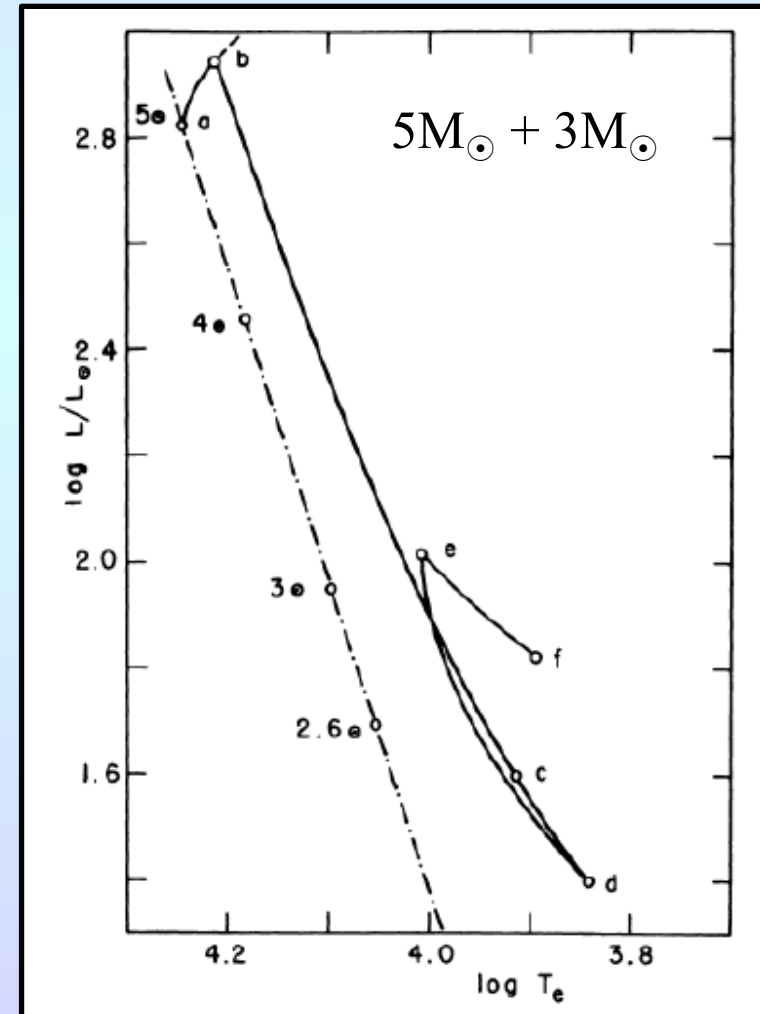
For stars with convective envelopes, the strength of their activity depends on their rotation rate. Tidally-locked stars can have star-spots that cover a significant fraction of their surface.



# Example: Main Sequence (Case A) Mass Transfer

In the case of two high-mass main sequence stars, the higher-mass ( $M_2$ ) will evolve first. Since  $q > 5/6$ , mass loss will occur on a dynamical timescale, reversing the mass ratio. Eventually,  $q$  will drop below  $5/6$ .

- Since  $q$  is now  $< 5/6$ , mass transfer slows to a nuclear timescale.
- Since  $M_2$  now has less mass, it will move down the main sequence as quickly as it can (on a thermal timescale.) The opposite is true for Star 2.
- Since  $M_1$  is now more massive, it will someday overflow its Roche Lobe, again making a contact system.

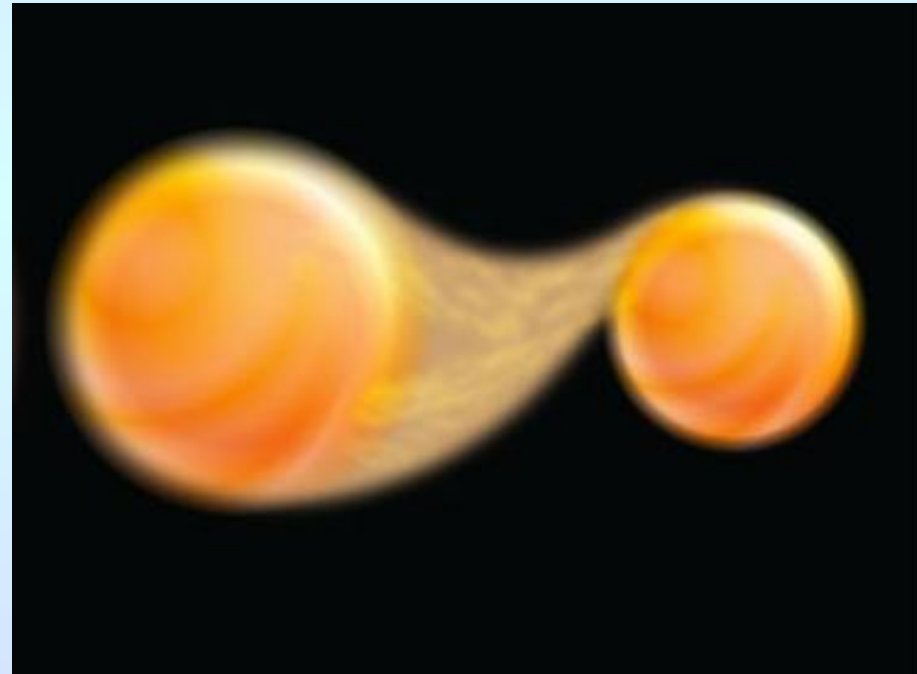


# Example: Main Sequence (Case A) Mass Transfer

The case of lower-mass main sequence stars is similar to that of the higher mass stars. However, because the stars have convective envelopes, magnetic breaking may also be important.

- **Blue stragglers** appear to rotate more rapidly than normal main-sequence stars, suggesting conversion of orbital angular momentum into spin.

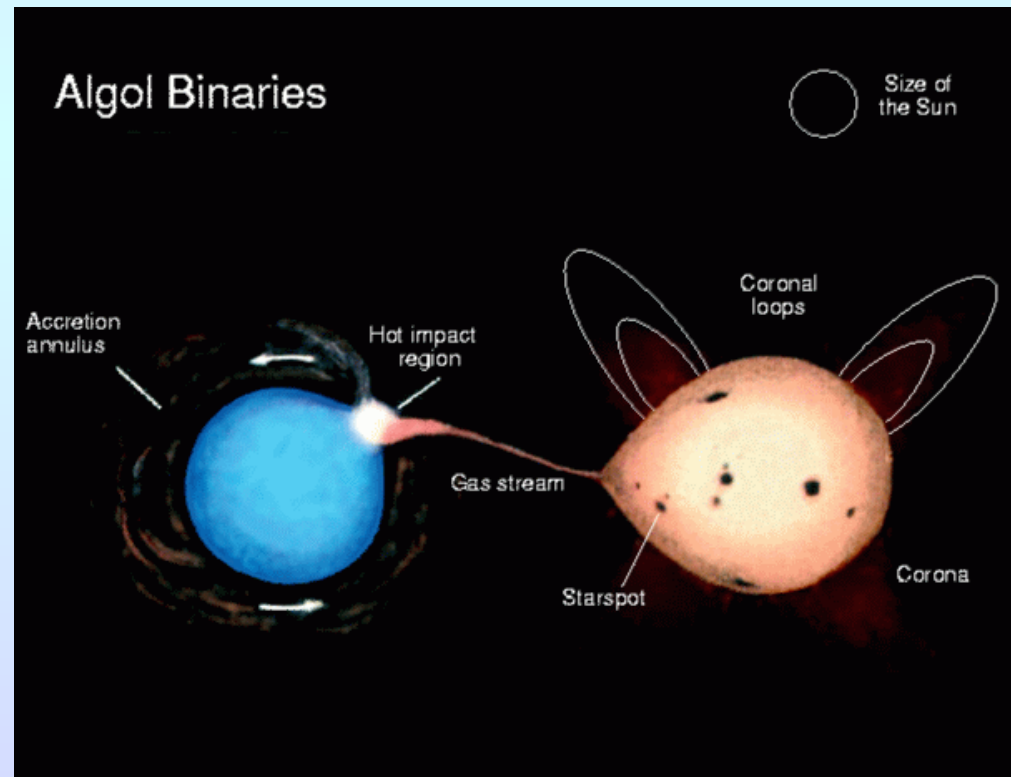
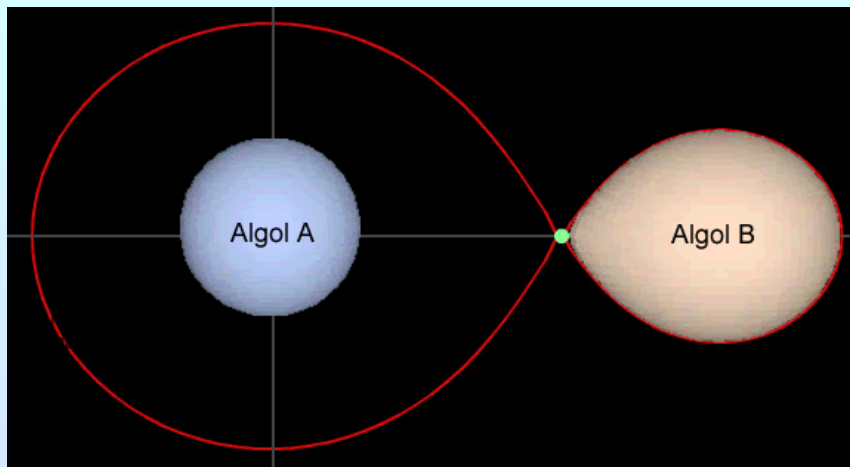
- Note: the creation of blue stragglers is probably more complicated than just binary star evolution. It may involve stellar collisions and/or three body interactions.



# Case A Close Binaries

**Algol Binaries:** Case A mass transfer has caused a reversal in the mass ratio of the stars, so that the system has a low-mass red subgiant star, and a high-mass main sequence star.

**W Serpens Stars:** Algol-type binaries with increased chromospheric activity from the giant.



# Case A Close Binaries

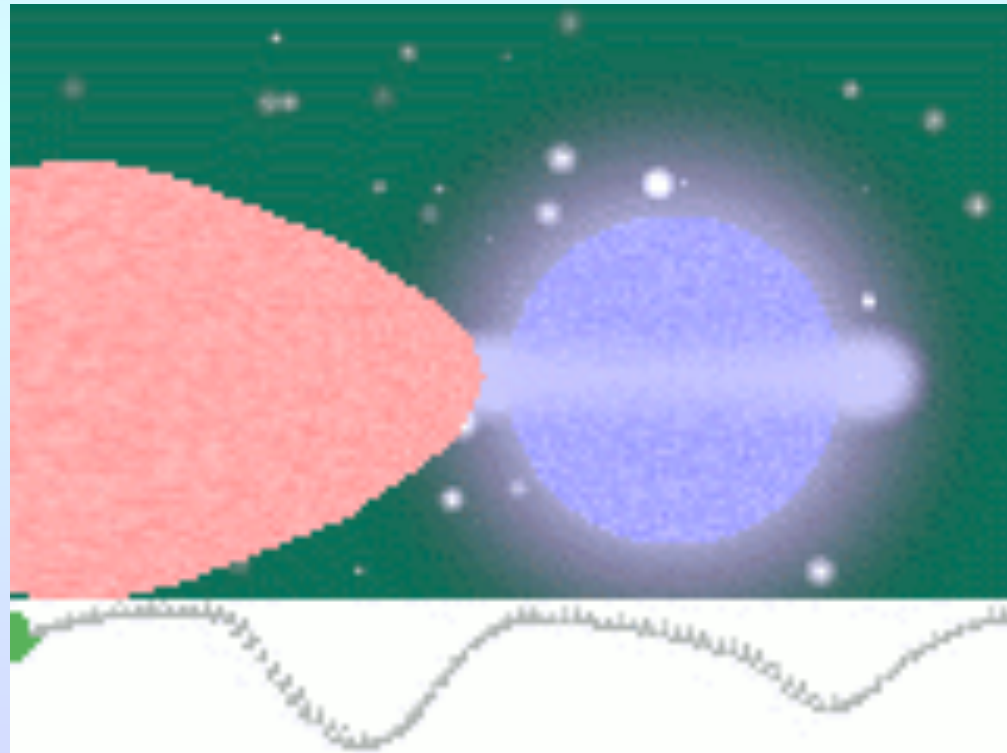
**Algol Binaries:** Case A mass transfer has caused a reversal in the mass ratio of the stars, so that the system has a low-mass red subgiant star, and a high-mass main sequence star.

**W Serpens Stars:** Algol-type binaries with increased chromospheric activity from the giant.

**RS Can Van and BY Draconis Stars:** Similar to Algols, but with lower-mass main sequence stars.

**Beta Lyrae Stars:** shorter period systems with more tidal distortion.

**W Ursae Majoris Stars:** contact binaries with very high chromospheric activity





# Case C Close Binaries

**Barium and s-process binaries:** long period binaries where the original high-mass star evolved to a white dwarf while transferring processed material to its now red giant companion.

**Zeta Aurigae Stars:** Long period G or K supergiant plus hot companion, which recently began mass transfer.

**VV Cepheid Stars:** Similar to Zeta Aurigae stars, except the supergiant is an M star.

**Symbiotic Stars:** Long period systems with a cool M giant (sometimes a pulsating Mira-type variable) accreting onto a hot companion such as a white dwarf or sub-dwarf.